Pink Kangaroo 2019

Solutions

1. **D** Note that $20 \times 19 + 20 + 19 = 20 \times 20 + 19 = 400 + 19 = 419$.

2. **B** Six circuits of 1 minute and 11 seconds take 6 minutes and 66 seconds. However, 66 seconds is 1 minute and 6 seconds, so the time taken is 7 minutes and 6 seconds.

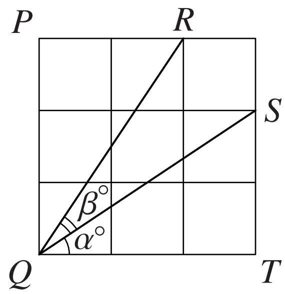
3. **E** The letters must appear in reverse order, EVAHS, and each letter must be reflected, so option E is correct.

4. **C** The smallest total that can be achieved is $1 + 1 + 1 = 3$, and the greatest is $6 + 6 + 6 = 18$. Every integer total in between can be obtained, so there are 16 possibilities.

5. **B** For each of the 5 ways in, there are 4 ways out, so there are $5 \times 4 = 20$ ways.

6. **D** Let J kg be the weight of the lightest kangaroo. Then the total weight of the three kangaroos is at least $3J$ kg. And we know they weigh 97 kg in total, so $3J \le 97$, so $J \le 32\frac{1}{3}$. Hence the lightest is at most 32 kg. The three could have weights 32 kg, 32 kg and 33 kg.

7. **B** The triangle PQR is congruent to the triangle TQS since they are right-angled triangles with sides of length 3 and 2. Hence the angle PQR is also $\alpha°$ and then $\alpha + \beta + \alpha = 90$. One can check that the other statements are false.



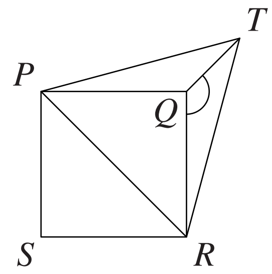
8. **A** In each square, each triangle has height one unit. In each of the squares B, C, D, E, the sum of the bases of these triangles is one unit since they cover one side of the unit square, so the shaded area is half of each unit square. However, in square A, there is a rectangle which covers double the area of a triangle of height 1 on the same base. Thus A has the largest shaded area.

9. **B** Let the missing digits be P, Q, R. Placing the numbers in a column addition, we get:

$$\begin{array}{rccccc} & 1 & 5 & 7 & 2 & 8 \\ & 2 & 2 & P & 0 & 4 \\ + & Q & R & 3 & 3 & 1 \\ \hline & 5 & 7 & 2 & 6 & 3 \end{array}$$

There is nothing to carry from the sum of the tens digits, so the sum of the middle digits is $7 + P + 3$ and this must end in 2. Hence $P = 2$. The sum of the digits in the next column is $5 + 2 + R + 1$ (where the 1 is carried from the middle digits). This must end in a 7, giving $R = 9$. The first digits have sum $1 + 2 + Q + 1$ and must end in 5 so $Q = 1$. Therefore the missing digits are 1, 2, 9.

10. C The diagram of the square and the triangle is shown. Consider the triangles PQT and RQT. They share side QT; also $PQ = RQ$ because they are sides of a square; and $PT = RT$ because they are sides of the equilateral triangle, so by SSS the triangles PQT and RQT are congruent. Therefore $\angle PQT = \angle RQT$. Also $\angle PQT + \angle RQT + 90° = 360°$ so $\angle RQT = \frac{1}{2}(360° - 90°) = 135°$.

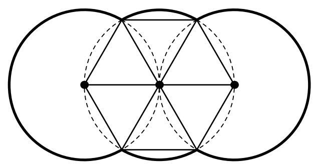


11. C To obtain the smallest value, the numerators should be as small as possible, so 1 or 2, and the denominators should be as large as possible, so 9 or 10. The two possible candidates are $\frac{1}{10} + \frac{2}{9}$ and $\frac{1}{9} + \frac{2}{10}$. The first gives $\frac{9}{90} + \frac{20}{90} = \frac{29}{90}$, and the second gives $\frac{10}{90} + \frac{18}{90} = \frac{28}{90} = \frac{14}{45}$, which is smaller.

12. E Let the length of the white rectangle be x and its height y. Then the height of the flag is $3y$ and hence its width is $5y$. The four rectangles which make up the flag are equal in area, so we have $3y \times 5y = 4xy$. This simplifies to $15y = 4x$ (since y is non-zero) and hence $y : x = 4 : 15$.

13. D The cycling and running account for $\frac{3}{4} + \frac{1}{5} = \frac{15}{20} + \frac{4}{20} = \frac{19}{20}$ of the distance. So the swimming, which is 2 km, is the remaining $\frac{1}{20}$ of the distance. Hence the distance is $20 \times 2 = 40$km.

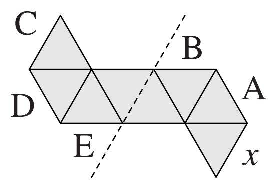
14. C By drawing radii and chords as shown, we can see that the triangles are equilateral and therefore each of the angles is 60°. Hence the left and right circles have each lost 120° (one third) of their circumferences, and the central circle has one-third $(\frac{1}{6} + \frac{1}{6})$ of its circumference remaining. Since $1 - \frac{1}{3} + 1 - \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$, the perimeter of the shape is $\frac{5}{3} \times 2\pi R = \frac{10\pi R}{3}$.



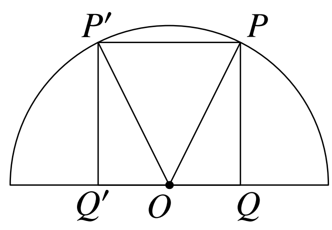
15. C The value of 'ab' is $10a + b$ so $3a + 4b = 10a + b$ which gives $3b = 7a$. Hence a is a multiple of 3 and b is a multiple of 7. Also a is non-zero (as it is the leading digit), and hence b also cannot be zero. The only single-digit solution of $3b = 7a$ is $a = 3$ and $b = 7$, so $a + b = 10$.

16. D Since each box has a different number of pears, and we want as many boxes as possible, we could start by putting no pears in box 1, 1 pear in box 2, 2 pears in box 3, and so on. By the 11th box we have $0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$ pears, and there are not enough pears for another box. However, we can't use 11 boxes because we need to share the 60 apples evenly and 60 is not a multiple of 11. So, the maximum possible is 10 boxes with 6 apples in each box, and the pears distributed as stated but with 24 in the 10th box.

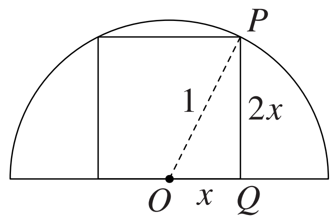
17. E The four triangles on the left will fold to form one square-based pyramid (without the base). The four triangles on the right will fold to make another pyramid, with the two pyramids hinged at the dashed edge. When these two pyramids are folded at this edge, the bottom end of x will coincide with the right-hand end of E; so x will coincide with E.



18. **A** Let O be the centre of the circle, and P, Q, P' and Q' the vertices of the square. The triangles OPQ and $OP'Q'$ are congruent since they are right-angled and have two equal sides ($PQ = P'Q'$ since they are edges of a square, and $OP = OP'$ because each is a radius). Hence $OQ = OQ'$, and O is thus the midpoint of the edge of the square.

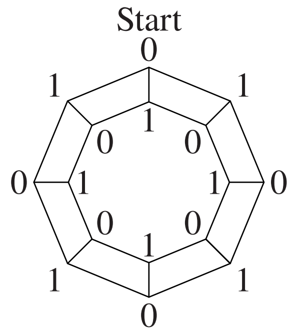


Let the side of the square be $2x$. And $OP = 1$ since it is a radius. And by Pythagoras' Theorem on triangle OPQ we have $(2x)^2 + x^2 = 1^2$, so $4x^2 + x^2 = 1$, and $5x^2 = 1$. This gives $x^2 = \frac{1}{5}$. The area of the square is $(2x)^2 = 4x^2 = 4 \times \frac{1}{5} = \frac{4}{5}$.



19. **D** The two-digit integers start with the pair of triples (101), (112). If we continue to consider the triples in pairs, then the following pairs start with the integers 13, 16, 19, etc. That is, they start with those integers that are 1 more than a multiple of 3. All of 22, 43, 46, and 88 have that form, they start the triples (222), (434), (464) and (888). Although 76 also has that form, it starts the triples (767) followed by (778), so that 777 is not a triple.

20. **C** Labelling vertices alternately 0/1 leads to the labelling shown. After an odd number of steps, the ant is always on a vertex labelled 1. The only such vertex labelled with a letter is Q.



21. **C** Let the digits of a be 'pqp', so $a = 101p + 10q$.Also $c = 2b + 1 = 2(2a + 1) + 1 = 4a + 3$ which is $404p + 40q + 3$. This is less than 1000, so $p = 1$ or 2.

If $p = 2$, then $c = 808 + 40q + 3$. This ends with digit 1 ($8 + 3 = 11$) but c is a 3-digit number greater than 808 so can't begin with 1.

If $p = 1$, then $c = 404 + 40q + 3$. This ends with digit $4 + 3 = 7$, so must also begin with 7, hence $700 \leq 404 + 40q + 3 < 800$ and thus $293 \leq 40q < 393$. Therefore $q = 8$ or 9.

When $q = 8$, this gives $a = 181$, $b = 363$, $c = 727$. When $q = 9$, this gives $a = 191$, $b = 383$, $c = 767$.

Hence there are two possibilities for a, 181 and 191.

22. **D** It is clear that none of the integers can be 1, since then the diagonally opposite integer will certainly be a multiple of it.

Let the smallest integer be a, and the diagonally opposite integer be b. Since a is the smallest integer, the other two numbers must both be multiples of a, say ma and na, for some integers m, n. Now b cannot be a multiple of a since a and b are diagonally opposite, so b cannot be a multiple of its neighbours ma and na; hence both ma and na are multiples of b.

Suppose a and b have a factor k in common (with $k > 1$). Then the other two integers also have this factor since they are multiples of a and of b. But then each of the four integers could be divided by this factor to produce four smaller integers with the desired properties. But we are looking for the smallest possible sum, hence a and b must not have any factors in common. The smallest pair of integers with no common factors to consider would be $a = 2$ and $b = 3$.

The other two integers must be multiples of both 2 and 3, but not of each other. We cannot use 6 because this would be a factor of any other multiple of both 2 and 3. The smallest possible pair is 12 and 18.

This gives a total of $2 + 3 + 12 + 18$ which is 35.

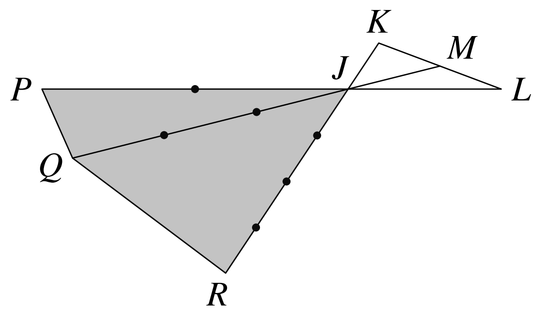
23. B To end up with a square product, any prime factor must occur an even number of times. Rhona cannot use 70 since the factor 7 only appears once in 70, and never in any of the other numbers.

If she uses all the other numbers she gets

$$\begin{aligned}&10 \times 20 \times 30 \times 40 \times 50 \times 60 \times 80 \times 90\\ &= (2 \times 5) \times (2 \times 2 \times 5) \times (2 \times 3 \times 5) \times (2 \times 2 \times 2 \times 5) \times (2 \times 5 \times 5)\\ &\quad \times (2 \times 2 \times 3 \times 5) \times (2 \times 2 \times 2 \times 2 \times 5) \times (2 \times 3 \times 3 \times 5)\\ &= 2^{15} \times 3^4 \times 5^9.\end{aligned}$$

To get a square, she needs to make the powers of 2 and 5 even, say by removing 10 from the list (although removing 40 or 90 works too). Hence she needs to remove only two numbers from her list.

24. A Let u be the area of triangle JLM, and v be the area of triangle JMK. Since M is the midpoint of KL, the triangle JLM has half the area of the triangle JLK, so $u = v = \frac{1}{2}S$.



Note that $\angle QJR = \angle KJM$, $JR = 4 \times JK$ and $JQ = 3 \times JM$. Hence, using the formula "Area of a triangle $= \frac{1}{2}ab\sin C$", we see that the area of triangle $JQR = 3 \times 4 \times v = 6S$.

Similarly the area of triangle $PJQ = 2 \times 3 \times u = 6u = 3S$ and the area of triangle $JPR = 2 \times 4 \times S = 8S$. Hence the area of triangle $PQR = 6S + 3S - 8S = S$.

25. C Let the 4-digit integer be '$abcd$'. When it is divided by 'abc', we get '$abcd$' $= 10 \times$ 'abc' $+ d$. Since 'abc' is a factor of '$abcd$', we must have $d = 0$, and the integer is '$abc0$'. Similarly, when '$abc0$' is divided by '$ab0$', we get '$abc0$' $= 10 \times$ '$ab0$' $+$ '$c0$'. But '$c0$' can only be divisible by '$ab0$' if $c = 0$. Thus the integer is '$ab00$'. This is divisible by '$b00$', so we can't have $b = 0$. When we divide '$ab00$' by '$a00$', we get '$ab00$' $= 10 \times$ '$a00$' $+$ '$b00$', hence '$b00$' must be a multiple of '$a00$', and therefore b is a multiple of a.

Since '$b00$' is a factor of '$ab00$' and '$ab00$' $=$ '$a000$' $+$ '$b00$', it must be that '$b00$' divides '$a000$', hence '$a0$' is a multiple of b. For b to be a multiple of a and a factor of $10 \times a$, b must be a, $2a$, $5a$ or $10a$. But $10a$ is more than one digit long. When $b = a$ we get '$ab00$' is 1100, 2200, 3300, 4400, 5500, 6600, 7700, 8800, 9900. When $b = 2a$, we get 1200, 2400, 3600, 4800. When $b = 5a$, we get 1500. This is $9 + 5 = 14$ possibilities.